Solutions to short-answer questions

1 a Employees of the company

b $p = \dfrac{\text{number of females in the company}}{\text{number of people in company}} = 0.35$

c $\hat{p} = \dfrac{\text{number of females in the sample}}{\text{number of people in the sample}} = 0.4$

2 No, this sample (people already interested in yoga) is not representative of the population

3 No, people who choose to live in houses with gardens may not be representative of the population

4 a People with Type II diabetes

b Population is too large and dispersed to use for such an experiment.

c Unknown

d $\bar{x} = 1.5$

5 a All of the employees of the company

b $p =$ number of people in the company who are tertiary qualified divided by the number of people in the company $= 0.2$

c $\hat{p}$ number of people in the sample who are tertiary qualified divided by the number of people in the sample $= 0.22$

6

a p =number of people in the team who are female divided by the number of people in the team = $\dfrac{3}{5}$

b The values of $\hat{P}$ are $\dfrac{1}{3}, \dfrac{2}{3}, 1$

c

$$\begin{aligned}\Pr(\hat{P} = \tfrac{1}{3}) &= \Pr(X = 1)\\ &= \frac{\binom{3}{1}\binom{2}{2}}{\binom{5}{3}}\\ &= \frac{3}{10}\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P} = \tfrac{2}{3}) &= \Pr(X = 2)\\ &= \frac{\binom{3}{2}\binom{2}{1}}{\binom{5}{3}}\\ &= \frac{6}{10}\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P} = 1) &= \Pr(X = 3)\\ &= \frac{\binom{3}{3}\binom{2}{0}}{\binom{5}{3}}\\ &= \frac{1}{10}\end{aligned}$$

$\hat{p}$	$\frac{1}{3}$	$\frac{2}{3}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

d $\Pr(\hat{P} > 0.5) = \frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

e $\Pr(0 < \hat{P} < 0.5) = \frac{3}{10}$, $\Pr(\hat{P} < 0.5 \mid \hat{P} > 0) = \frac{3}{10}$

7 a Values of $\hat{P}$ are $0, 0.25, 0.5, 0.75, 1$

b Binomial with $n = 4, p = 0.5$

$$\begin{aligned}\Pr(\hat{P} = 0) &= \Pr(X = 0)\\ &= \binom{4}{0}(0.5)^0(0.5)^4\\ &= 0.0625\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P} = 0.25) &= \Pr(X = 1)\\ &= \binom{4}{1}(0.5)^1(0.5)^3\\ &= 0.25\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P} = 0.5) &= \Pr(X = 2)\\ &= \binom{4}{2}(0.5)^2(0.5)^2\\ &= 0.375\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P} = 0.75) &= \Pr(X = 3)\\ &= \binom{4}{3}(0.5)^3(0.5)^1\\ &= 0.25\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P} = 1) &= \Pr(X = 4)\\ &= \binom{4}{4}(0.5)^4(0.5)^0\\ &= 0.0625\end{aligned}$$

$\hat{p}$	0	0.25	0.5	0.75	1
$\Pr(\hat{P} = \hat{p})$	0.0625	0.25	0.375	0.25	0.0625

c $\Pr(\hat{P} < 0.5) = 0.3125$

d
$$\begin{aligned}\Pr(\hat{P} < 0.5 \mid \hat{P} < 0.8) &= \frac{\Pr(\hat{P} < 0.5)}{\Pr(\hat{P} < 0.8)}\\ &= \frac{1}{3}\end{aligned}$$

8 a i There are three dots which represent sample proportions of 0.7 or more fromthe 100 samples simulated. Thus we can estimate $\Pr(\hat{P} \geq 0.7) = 0.03$

ii There are four dots which represent a sample proportion of 0.38 or less fromthe 100 samples simulated.Thus we can estimate $\Pr(\hat{P} \leq 0.38) = 0.04$

b i $\hat{p} = \frac{\text{number of people in the sample who will vote for Bill Bloggs}}{\text{number of people in the sample}} = 0.42$

ii From the plot there are 8 samples where the sample proportion is 0.42 or less,from the 100 simulations. Thus we can estimate that $\Pr(\hat{P} \leq 0.42) = 0.08$

Solutions to multiple-choice questions

1 **B** Since this ratio is determined from a sample it is a sample statistic.

2 **C** Since this percentage is determined from complete census it is a population parameter

3 **A** In reality we rarely know the value of a population parameter, whereas we can determine the value of a sample statistic. So generally, we are using the sample statistic to estimate the value of a population parameter.

4 **B** The sampling distribution describes how the values of the sampling distribution vary from sample to sample.

5 **B** Since sample statistics are estimates of population parameters.

6 **E**

$$\begin{aligned}\Pr(\hat{P} \geq 0.7|\hat{P} > 0.2) &= \frac{\Pr(\hat{P} \geq 0.7)}{\Pr(\hat{P} > 0.2)} \\ &= \frac{(0.048 + 0.005}{(0.572} \\ &= \frac{0.053}{0.572} \\ &= 0.092657\ldots\end{aligned}$$

7 **A** There are 4 vegetarians and 6 non-vegetarians. If the proportion of vegetarians with plastic plates is $\hat{P}$, then:
$\hat{P}$ takes values $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$
If $\hat{P} = \frac{3}{4}$ then three of the 4 vegetarians got a plastic plate, and 4 of the 6 non-vegetarians got a plastic plate, that is:

$$\begin{aligned}\Pr(\hat{P} = 1) &= \frac{\binom{4}{4}\binom{6}{0}}{\binom{10}{4}} \\ &= 0.1143\end{aligned}$$

and

$$\begin{aligned}\Pr(\hat{P} = \tfrac{3}{4}) &= \frac{\binom{4}{3}\binom{6}{1}}{\binom{10}{4}} \\ &= 0.0048\end{aligned}$$

Thus $\Pr(\hat{P} > 0.5) = 0.1191$

8 **D** There are 12 gold fish and 8 black fish. If the proportion of gold fish in the sample of five is $\hat{p}$, then $\hat{P}$ takes values 0, 0.2, 0.4, 0.6, 0.8, 1.
If $\hat{P} = 0.8$, then 4 of the 5 fish are gold, that is:

$$\begin{aligned}\Pr(\hat{P} = 0.8) &= \frac{\binom{12}{4}\binom{8}{1}}{\binom{20}{5}} \\ &= 0.2554\end{aligned}$$

and

$$\begin{aligned}\Pr(\hat{P} = 1) &= \frac{\binom{12}{5}\binom{8}{0}}{\binom{20}{5}} \\ &= 0.0511\end{aligned}$$

Thus $\Pr(\hat{P} > 0.8) = 0.3065$

9 **E** $X =$ number of students who study Chinese in the sample of size 20.
X has a binomial distribution, $n = 20, p = 0.2$
$\hat{P}$ = proportion of students who speak Chinese in the sample of size 20. $\hat{P}$ takes values 0, 0.05, 0.1, 0.15, 0.20, 0.951

$\Pr(\hat{P} < 0.1) = \Pr(\hat{P} = 0) + \Pr(\hat{P} = 0.05)$
We have
$\Pr \hat{P} = 0) = \Pr(X = 0) = 0.0015$
$\Pr(\hat{P} = 0.05) = \Pr(X = 1) = 0.0576$
$\therefore \Pr(\hat{P} < 0.10) = 0.0691$

10 **B** $X =$ number of heads in the sample of size 10.
X has a binomial distribution, $n = 10, p = 0.5$
$\hat{P}$ = proportion of heads in the sample of size 10.
$\hat{P}$ takes values $0, 0.1, 0.2, 0.3 \ldots$
$\Pr(\hat{P} < 0.2 \text{ or } \hat{P} > 0.8) = \Pr(\hat{P} < 0.2) + \Pr(\hat{P} > 0.8)$

$\Pr(\hat{P} < 0.2) = \Pr(X < 2) = 0.01074$
$\Pr(\hat{P} > 0.8) = \Pr(X > 8)$ = 0.01074
$\Pr(\hat{P} < 0.2$ or $\Pr \hat{P} > 0.8) = 0.02148 \approx 2\%$

11 **C** The exact sampling distribution is hypergeometric, but when the sample size is small compared to the population size, the binomial distribution gives a good approximation.

12 **E** Thus increasing the sample size will result in a decrease in the variability of the sample estimates, as we have seen from the sampling dsitributions.

Solutions to extended-response questions

1 **a**

p	a	b
0.1	0.03	0.17
0.2	0.11	0.29
0.3	0.19	0.41
0.4	0.29	0.51
0.5	0.38	0.62
0.6	0.49	0.71

b **i** $\hat{p} = 0.34$

ii $p = 0.3$ or $p = 0.4$

2 **a** **iii** mean ≈ 50, s.d. ≈ 1.12

b **iii** mean ≈ 50, s.d. ≈ 0.71

c **iii** mean ≈ 50, s.d. ≈ 0.50